Centre de Physique Théorique*, **CNRS Luminy, Case 907**

F-13288 Marseille – Cedex 9

N=4 Supergravity with Antisymmetric Tensor in Central Charge Superspace

Richard GRIMM [1], **Carl HERRMANN** [2] **and Annamária KISS** [1]

[1] *Centre de Physique Théorique, CNRS Luminy Case 907, F–13288 Marseille Cedex 9, France*

[2] *Fachgruppe Theoretische Physik - FB Physik, Martin Luther Universität Halle–Wittenberg, D–06099 Halle, Germany*

Abstract

A concise geometrical formulation of $N = 4$ supergravity containing an antisymmetric tensor gauge field is given in central charge superspace: graviphotons are identified in the super–vielbein on the same footing as the vierbein and the Rarita–Schwinger fields. As a consequence of superspace soldering, Chern–Simons terms in the fieldstrength of the antisymmetric tensor arise as an intrinsic property of superspace with central charge coordinates.

Key-Words: extended supersymmetry, central charge superspace.

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anonymous ftp : ftp.cpt.univ-mrs.fr

web : www.cpt.univ-mrs.fr

*Unité Propre de Recherche 7061

1 Introduction

$N = 4$ is the smallest extended supergravity theory which contains on–shell helicity 0 states. There are two versions of $N = 4$ supergravity which realise both of the two helicity 0 states as a complex scalar: one with an off–shell $SO(4)$ and an on–shell $SU(4)$ global symmetry and another one with a global off-shell $SU(4)$ symmetry [1], [2], [3], [4], [5], [6]. A third version, containing a real scalar and an antisymmetric tensor gauge field as helicity 0 states in its spectrum [7], has been formulated in terms of component fields by Nicolai and Townsend [8]. The latter, which we will call the N–T multiplet in the sequel, is related to the previous theory by a scalar-tensor duality transformation.

As to geometric formulations in superspace [9], [10], [11], the former, containing a complex scalar with $SU(1,1)/U(1)$ coset structure, has been given both in canonical superspace [11] and in superspace extended by bosonic coordinates corresponding to central charges [10]. On the other hand, the superspace formulation of the N–T multiplet encountered a number of problems [11], [12]. We present here a concise central charge superspace formulation, which allows to derive the N–T theory in a natural and straightforward way.

Recall that in central charge geometry, the frame of superspace $E^{\mathcal{A}}$ has components $E^{\mathcal{A}} = (E^a, E^\alpha_{\mathrm{A}}, E_{\dot\alpha}^{\mathrm{A}}, E^{\mathbf{u}})$. Here a, α, $\dot\alpha$ denote the usual vector and Weyl spinor indices while capital indices A count the number of supercharges and boldface indices $\mathbf{u}$ the number of central charges. This framework provides a unified geometric identification of graviton, gravitini *and* graviphotons as lowest superfield components such that

$$E^a\| \;=\; dx^m e_m{}^a \;, \quad E^\alpha_{\mathrm{A}}\| \;=\; \frac{1}{2}\, dx^m \psi_m{}^\alpha_{\mathrm{A}} \;, \quad E_{\dot\alpha}^{\mathrm{A}}\| \;=\; \frac{1}{2}\, dx^m \bar\psi_m{}^{\mathrm{A}}_{\dot\alpha} \;, \quad E^{\mathbf{u}}\| \;=\; dx^m v_m{}^{\mathbf{u}} \;. \tag{1.1}$$

The double bar projects at the same time on the vector coefficient of the differential form and on the lowest superfield component. On the other hand, the antisymmetric tensor is identified in a superspace 2–form B such that

$$B\| \;=\; \frac{1}{2}\, dx^m dx^n b_{nm}. \tag{1.2}$$

At this stage we still have two separate geometric structures: the supergravity sector and the 2–form sector. In order to identify b_{mn} in the same multiplet as $e_m{}^a$, $\psi_m{}^\alpha_{\mathrm{A}}$, $\bar\psi_m{}^{\mathrm{A}}_{\dot\alpha}$, $v_m{}^{\mathbf{u}}$ we will use a mechanism called superspace soldering, already known in other contexts of superspace geometry. This mechanism relies crucially on the special form of certain torsion coefficients. In our case they are

$$T_{\gamma\beta}^{\mathrm{CB}a} \;=\; 0 \;, \qquad T_{\gamma\mathrm{B}}^{\mathrm{C}\dot\beta a} \;=\; -2i\delta^{\mathrm{C}}_{\mathrm{B}}(\sigma^a\epsilon)_\gamma{}^{\dot\beta} \;, \qquad T^{\dot\gamma\dot\beta a}_{\mathrm{CB}} \;=\; 0 \;, \tag{1.3}$$

$$T_{\gamma\beta}^{\mathrm{CB}\mathbf{u}} \;=\; \epsilon_{\gamma\beta} T^{[\mathrm{CB}]\mathbf{u}} \;, \qquad T_{\gamma\ \mathrm{B}}^{\mathrm{C}\dot\beta\mathbf{u}} \;=\; 0 \;, \qquad T^{\dot\gamma\dot\beta\mathbf{u}}_{\mathrm{CB}} \;=\; \epsilon^{\dot\gamma\dot\beta} T_{[\mathrm{CB}]}{}^{\mathbf{u}} \;. \tag{1.4}$$

As we will show below, the remaining components of the multiplet, the real scalar and the helicity 1/2 fields, will be identified in torsion components. Whereas the real scalar will appear in $T^{[\mathrm{CB}]\mathbf{u}}$ and $T_{[\mathrm{CB}]}{}^{\mathbf{u}}$, the helicity 1/2 fields, which we shall call *gravigini* fields, are identified in the lowest superfield components of

$$T_{\gamma\ \beta\dot\alpha}^{\mathrm{CBA}} \;=\; \epsilon_{\gamma\beta} T^{[\mathrm{CBA}]}{}_{\dot\alpha} \;, \qquad T^{\dot\gamma\dot\beta\alpha}_{\mathrm{C\,BA}} \;=\; \epsilon^{\dot\gamma\dot\beta} T_{[\mathrm{CBA}]}{}^{\alpha} \;. \tag{1.5}$$

The scalar, the helicity 1/2 gravigino together with the component fields defined in (1.1) and (1.2) constitute the N–T multiplet.

2 General geometric superspace structures

As mentioned above, the basic ingredients in our construction of $N = 4$ supergravity are the local frame $E^{\mathcal{A}}$ in superspace and the 2–form gauge potential B.

Superspace diffeomorphisms, which now contain general space–time coordinate transformations, local supersymmetry transformations as well as local central charge transformations, are implemented in the usual way. Covariant derivatives with respect to structure group transformations are defined in terms of the connection 1–form $\Phi_{\mathcal{B}}{}^{\mathcal{A}}$. Torsion, curvature and fieldstrength of the 2–form gauge potential are given as

$$T^{\mathcal{A}} \;=\; DE^{\mathcal{A}} \;=\; dE^{\mathcal{A}} + E^{\mathcal{B}}\Phi_{\mathcal{B}}{}^{\mathcal{A}} \;, \tag{2.1}$$

$$R_{\mathcal{B}}{}^{\mathcal{A}} \;=\; d\Phi_{\mathcal{B}}{}^{\mathcal{A}} + \Phi_{\mathcal{B}}{}^{\mathcal{C}}\Phi_{\mathcal{C}}{}^{\mathcal{A}} \;, \tag{2.2}$$

$$H \;=\; dB \;. \tag{2.3}$$

Following standard textbook procedures [13] we introduce supergravity, or Wess-Zumino [14] transformations

$$\delta_{\rm wz} E^{\mathcal{A}} \;=\; D\xi^{\mathcal{A}} + \imath_\xi T^{\mathcal{A}} \;, \tag{2.4}$$

$$\delta_{\rm wz} B \;=\; \imath_\xi H \;, \tag{2.5}$$

$$\delta_{\rm wz} \Phi_{\mathcal{B}}{}^{\mathcal{A}} \;=\; \imath_\xi R_{\mathcal{B}}{}^{\mathcal{A}} \;, \tag{2.6}$$

as certain combinations of superspace diffeomorphisms and structure group transformations. Observe that the complete structure of commutation relations derives in a constructive way. For later convenience we display here the explicit form of the transformation of the vielbein and of the 2–form:

$$\delta_{\rm wz}\, E_{\mathcal{M}}{}^{\mathcal{A}} \;=\; \mathcal{D}_{\mathcal{M}}\xi^{\mathcal{A}} + E_{\mathcal{M}}{}^{\mathcal{B}}\,\xi^{\mathcal{C}} T_{\mathcal{C}\mathcal{B}}{}^{\mathcal{A}} \;, \tag{2.7}$$

$$\delta_{\rm wz}\, B_{\mathcal{N}\mathcal{M}} \;=\; (-)^{n(m+a)} E_{\mathcal{M}}{}^{\mathcal{A}} E_{\mathcal{N}}{}^{\mathcal{B}}\,\xi^{\mathcal{C}} H_{\mathcal{C}\mathcal{B}\mathcal{A}} \;. \tag{2.8}$$

Observe that on the level of component fields, *i.e.* lowest superfield values (1.1), (1.2), the transformations of parameters $\xi^{\alpha}_{\rm A}| = \zeta^{\alpha}_{\rm A}$ and $\xi^{\rm A}_{\dot\alpha}| = \bar\zeta^{\rm A}_{\dot\alpha}$ reproduce the Wess–Zumino gauge, justifying the notation $\delta_{\rm wz}$.

In our analysis, we will make systematic use of the torsion and 3–form Bianchi identities

$$DT^{\mathcal{A}} \;=\; E^{\mathcal{B}} R_{\mathcal{B}}{}^{\mathcal{A}} \;, \qquad dH \;=\; 0 \;. \tag{2.9}$$

In some more detail, we shall use the notation

$$\left({}_{\mathcal{D}\mathcal{C}\mathcal{B}}{}^{\mathcal{A}}\right)_T \qquad E^{\mathcal{B}} E^{\mathcal{C}} E^{\mathcal{D}} \left(\mathcal{D}_{\mathcal{D}} T_{\mathcal{C}\mathcal{B}}{}^{\mathcal{A}} + T_{\mathcal{D}\mathcal{C}}{}^{\mathcal{F}} T_{\mathcal{F}\mathcal{B}}{}^{\mathcal{A}} - R_{\mathcal{D}\mathcal{C}\mathcal{B}}{}^{\mathcal{A}}\right) \;=\; 0 \;, \tag{2.10}$$

$$\left({}_{\mathcal{D}\mathcal{C}\mathcal{B}\mathcal{A}}\right)_H \qquad E^{\mathcal{A}} E^{\mathcal{B}} E^{\mathcal{C}} E^{\mathcal{D}} \left(2\,\mathcal{D}_{\mathcal{D}} H_{\mathcal{C}\mathcal{B}\mathcal{A}} + 3\, T_{\mathcal{D}\mathcal{C}}{}^{\mathcal{F}} H_{\mathcal{F}\mathcal{B}\mathcal{A}}\right) \;=\; 0 \;, \tag{2.11}$$

displaying explicitly the 3–form and 4–form coefficients of the above relations.

3 Torsion, 3–form curl and superspace soldering

Before presenting in detail the superspace geometry which describes the N–T multiplet, we shall explain qualitatively the mechanism of superspace soldering. Roughly speaking, this operation allows to identify various components of one and the same supergravity multiplet in two distinct geometric structures: graviton $e_m{}^a$, gravitini $\psi_m{}^\alpha_{\mathrm{A}}$, $\bar\psi_m{}^{\mathrm{A}}_{\dot\alpha}$, and graviphotons $v_m{}^{\mathbf{u}}$ in the gravity sector, and the antisymmetric tensor b_{mn} in the 2–form sector.

The basic idea is to establish relations between the coefficients of the 3–form curl H and the torsion $T^{\mathcal{A}}$ in suitably choosing constraints in both sectors. As examples we will describe how H_{cba} will appear in the torsion coefficients, on the one hand, and the relation between $T_{cb}{}^{\mathbf{u}}$, the graviphoton fieldstrength and the coefficient $H_{\mathbf{u}ba}$ on the other hand. This will then lead automatically to the appearance of the graviphoton Chern–Simons form in the supercovariant component fieldstrength $H_{cba}|$.

To see intuitively how the soldering procedure works we will have a look at certain Bianchi identities. As to the relation of H_{cba} to torsions we consider $\binom{\mathrm{D}\dot\gamma}{\delta\mathrm{C}ba}_H$ in (2.11). At an early intermediate stage of the analysis, with some suitably chosen constraints, this superfield equation takes the form

$$
\begin{aligned}
&\mathcal{D}^{\mathrm{D}}_\delta H^{\dot\gamma}_{\mathrm{C}ba} + \mathcal{D}^{\dot\gamma}_{\mathrm{C}} H^{\mathrm{D}}_{\delta ba} + \mathcal{D}_b H^{\mathrm{D}\dot\gamma}_{\delta\mathrm{C}a} - \mathcal{D}_a H^{\mathrm{D}\dot\gamma}_{\delta\mathrm{C}b} \\
&+T^{\mathrm{D}\dot\gamma\varphi}_{\delta\mathrm{C}\mathrm{F}} H^{\mathrm{F}}_{\varphi ba} + T^{\mathrm{D}\dot\gamma\mathrm{F}}_{\delta\mathrm{C}\dot\varphi} H^{\dot\varphi}_{\mathrm{F}ba} + T^{\mathrm{D}}_{\delta a}{}^{\mathbf{u}} H_{\mathbf{u}}{}^{\dot\gamma}_{\mathrm{C}b} + T^{\dot\gamma}_{\mathrm{C}a}{}^{\mathbf{u}} H_{\mathbf{u}}{}^{\mathrm{D}}_{\delta b} - T^{\mathrm{D}}_{\delta b}{}^{\mathbf{u}} H_{\mathbf{u}}{}^{\dot\gamma}_{\mathrm{C}a} - T^{\dot\gamma}_{\mathrm{C}b}{}^{\mathbf{u}} H_{\mathbf{u}}{}^{\mathrm{D}}_{\delta a} \\
&+T^{\mathrm{D}\dot\gamma f}_{\delta\mathrm{C}} H_{fba} + T^{\mathrm{D}}_{\delta a}{}^{\varphi}_{\mathrm{F}} H^{\mathrm{F}\dot\gamma}_{\varphi\mathrm{C}b} + T^{\dot\gamma}_{\mathrm{C}a}{}^{\mathrm{F}}_{\dot\varphi} H^{\dot\varphi\mathrm{D}}_{\mathrm{F}\delta b} - T^{\mathrm{D}}_{\delta b}{}^{\varphi}_{\mathrm{F}} H^{\mathrm{F}\dot\gamma}_{\varphi\mathrm{C}a} - T^{\dot\gamma}_{\mathrm{C}b}{}^{\mathrm{F}}_{\dot\varphi} H^{\dot\varphi\mathrm{D}}_{\mathrm{F}\delta a} \ = \ 0 \ . \qquad (3.1)
\end{aligned}
$$

Although it contains a lot of information, this relation is less complicated than it seems to be. As it will be shown later on, the spinorial coefficients, *i.e.* $H^{\mathrm{C}}_{\gamma ba}$, $H^{\dot\gamma}_{\mathrm{C}ba}$, $T^{\mathrm{C}}_{\gamma b}{}^{\mathbf{u}}$, $T^{\dot\gamma}_{\mathrm{C}b}{}^{\mathbf{u}}$, $T^{\mathrm{C}\dot\beta\mathrm{A}}_{\gamma\mathrm{B}\dot\alpha}$, $T^{\dot\gamma\mathrm{B}\alpha}_{\mathrm{C}\beta\mathrm{A}}$, $H_{\mathbf{u}b}{}^{\mathrm{A}}_{\alpha}$, $H_{\mathbf{u}b}{}^{\dot\alpha}_{\mathrm{A}}$ are all expressed in terms of one and the same gravigino superfield (which contains the helicity 1/2 field of the multiplet in its lowest component), as a consequence of the Bianchi identities at dimension 1/2. The soldering is achieved in requiring

$$
T^{\mathrm{C}\dot\beta a}_{\gamma\mathrm{B}} \ = \ -2i\,\delta^{\mathrm{C}}_{\mathrm{B}}\,(\sigma^a\epsilon)_\gamma{}^{\dot\beta} \ , \qquad H^{\mathrm{C}\dot\beta}_{\gamma\mathrm{B}a} \ = \ -2i\,L\,\delta^{\mathrm{C}}_{\mathrm{B}}\,(\sigma_a\epsilon)_\gamma{}^{\dot\beta} \ , \qquad (3.2)
$$

with the superfield L pertaining to the real scalar of the theory. In the last line of (3.1) this gives rise to

$$
\begin{aligned}
&T^{\mathrm{D}\dot\gamma f}_{\delta\mathrm{C}} H_{fba} - T^{\mathrm{D}}_{\delta b}{}^{\varphi}_{\mathrm{F}} H^{\mathrm{F}\dot\gamma}_{\varphi\mathrm{C}a} - T^{\dot\gamma}_{\mathrm{C}b}{}^{\mathrm{F}}_{\dot\varphi} H^{\dot\varphi\mathrm{D}}_{\mathrm{F}\gamma a} \ = \\
&\qquad -2i\,\delta^{\mathrm{D}}_{\mathrm{C}}\,(\sigma^f\epsilon)_\delta{}^{\dot\gamma} H_{fba} + 2i\,L\,T^{\mathrm{D}}_{\delta b}{}^{\gamma}_{\mathrm{C}}\,(\sigma_a\epsilon)_\gamma{}^{\dot\gamma} + 2i\,L\,T^{\dot\gamma}_{\mathrm{C}b}{}^{\mathrm{D}}_{\dot\delta}(\bar\sigma_a\epsilon)^{\dot\delta}{}_\delta \ . \qquad (3.3)
\end{aligned}
$$

This qualitative discussion should give an idea in which way relations between H_{cba}, $\mathcal{D}_a L$ and $T^{\mathrm{C}}_{\gamma b}{}^{\mathrm{A}}_{\alpha}$, $T^{\mathrm{C}}_{\dot\gamma b}{}^{\dot\alpha}_{\mathrm{A}}$ will arise. It should also become clear that the full explicit analysis still requires a certain amount of computational efforts. Observe that, as a consequence of the above and other relations, $H_{cba}|$ and $\mathcal{D}_a L|$ will appear in the supergravity transformation laws of $\psi_m{}^\alpha_{\mathrm{A}}$, $\bar\psi_m{}^{\mathrm{A}}_{\dot\alpha}$, through $T^{\mathrm{C}}_{\gamma b}{}^{\alpha}_{\mathrm{A}}|$, $T^{\mathrm{C}}_{\dot\gamma b}{}^{\mathrm{A}}_{\dot\alpha}|$, as identified in (2.7).

In a similar way, relations between $T_{cb}{}^{\mathbf{u}}$, the graviphoton fieldstrength, and $H_{\mathbf{u}ba}$ as well as $T^{\mathrm{C}}_{\gamma b}{}^{\mathrm{A}}_{\dot\alpha}$, $T^{\dot\gamma}_{\mathrm{C}b}{}^{\alpha}_{\mathrm{A}}$ can be obtained. To illustrate this we consider the Bianchi identity

$\left({}^{\mathrm{DC}}_{\delta\gamma\, ba}\right)_H$ of (2.11) in the 2–form sector:

$$
\begin{aligned}
&\mathcal{D}^{\mathrm{D}}_{\delta} H^{\mathrm{C}}_{\gamma\, b\, a} + \mathcal{D}^{\mathrm{C}}_{\gamma} H^{\mathrm{D}}_{\delta\, b\, a} \\
&+ T_{\delta\gamma\mathrm{F}}^{\mathrm{D\,C}\varphi} H^{\mathrm{F}}_{\varphi\, b\, a} + T_{\delta\gamma\dot\varphi}^{\mathrm{D\,C\,F}} H^{\dot\varphi}_{\mathrm{F}\, b\, a} + T^{\mathrm{D}}_{\delta\, a}{}^{\mathbf{u}} H_{\mathbf{u}\,\gamma\, b}{}^{\mathrm{C}} + T^{\mathrm{C}}_{\gamma\, a}{}^{\mathbf{u}} H_{\mathbf{u}\,\delta\, b}{}^{\mathrm{D}} - T^{\mathrm{D}}_{\delta\, b}{}^{\mathbf{u}} H_{\mathbf{u}\,\gamma\, a}{}^{\mathrm{C}} - T^{\mathrm{C}}_{\gamma\, b}{}^{\mathbf{u}} H_{\mathbf{u}\,\delta\, a}{}^{\mathrm{D}} \\
&+ T^{\mathrm{D\,C}\,\mathbf{u}}_{\delta\gamma} H_{\mathbf{u}\, b\, a} + T_{b\, a}{}^{\mathbf{u}} H_{\mathbf{u}\,\delta\gamma}{}^{\mathrm{D\,C}} + T^{\mathrm{D}}_{\delta\, a}{}^{\mathrm{F}}_{\dot\varphi} H^{\dot\varphi\,\mathrm{C}}_{\mathrm{F}\,\gamma\, b} + T^{\mathrm{C}}_{\gamma\, a}{}^{\mathrm{F}}_{\dot\varphi} H^{\dot\varphi\,\mathrm{D}}_{\mathrm{F}\,\delta\, b} - T^{\mathrm{D}}_{\delta\, b}{}^{\mathrm{F}}_{\dot\varphi} H^{\dot\varphi\,\mathrm{C}}_{\mathrm{F}\,\gamma\, a} - T^{\mathrm{C}}_{\gamma\, b}{}^{\mathrm{F}}_{\dot\varphi} H^{\dot\varphi\,\mathrm{D}}_{\mathrm{F}\,\delta\, a} \;=\; 0\ .
\end{aligned}
$$

Again, we discard for the moment the discussion of the derivative and quadratic spinor terms. The terms pertinent for the soldering are those containing $T_{cb}{}^{\mathbf{u}}$, $H_{\mathbf{u}\, b\, a}$ and $T^{\mathrm{C}}_{\gamma\, b}{}^{\mathrm{A}}_{\dot\alpha}$. Here, as a second set of soldering conditions, we take

$$
T^{\mathrm{C\,B}\,\mathbf{u}}_{\gamma\,\beta} \;=\; \epsilon_{\gamma\beta}\, T^{[\mathrm{CB}]\mathbf{u}}\ , \qquad H_{\mathbf{u}\,\beta\,\alpha}{}^{\mathrm{B\,A}} \;=\; \epsilon_{\beta\alpha}\, H_{\mathbf{u}}{}^{[\mathrm{BA}]}\ , \tag{3.4}
$$

and correspondingly for the complex conjugates. The relevant terms in the above identity are

$$
T^{\mathrm{C\,B}\,\mathbf{u}}_{\gamma\,\beta} H_{\mathbf{u}\, b\, a} + T_{b\, a}{}^{\mathbf{u}} H_{\mathbf{u}\,\delta\,\gamma}{}^{\mathrm{D\,C}} \;=\; \epsilon_{\delta\gamma}\left(T^{[\mathrm{DC}]\mathbf{u}} H_{\mathbf{u}\, b\, a} + T_{b\, a}{}^{\mathbf{u}} H_{\mathbf{u}}{}^{[\mathrm{DC}]}\right)\ , \tag{3.5}
$$

as well as

$$
T^{\mathrm{D}}_{\delta\, b}{}^{\mathrm{F}}_{\dot\varphi}\, H^{\dot\varphi\,\mathrm{C}}_{\mathrm{F}\,\gamma\, a} \;=\; -2i\, L\, T^{\mathrm{D}}_{\delta\, b}{}^{\mathrm{C}}_{\dot\gamma}\, (\bar\sigma_a \epsilon)^{\dot\gamma}{}_{\gamma}\ . \tag{3.6}
$$

This should give an idea how, after some more computational efforts, $T^{[\mathrm{DC}]\mathbf{u}} H_{\mathbf{u}ba}$ as well as $T^{\mathrm{C}}_{\gamma\, b}{}^{\mathrm{A}}_{\dot\alpha}$ will be related to $T_{cb}{}^{\mathbf{u}}$, the graviphoton fieldstrength, with $T^{[\mathrm{DC}]\mathbf{u}}$ and $H_{\mathbf{u}}{}^{[\mathrm{DC}]}$ acting as converters between the central charge basis (indices $\mathbf{u}$, $\mathbf{v}$) and the $SU(4)$ basis in the antisymmetric representation (indices $[\mathrm{DC}]$). A more detailed analysis of the relevant Bianchi identities shows that $T_{cb}{}^{\mathbf{u}}$ will appear in the tensor decomposition of the torsion $T^{\mathrm{D}}_{\delta\, a}{}^{\mathrm{F}}_{\dot\varphi}$ and in the derivative terms $\mathcal{D}^{\mathrm{D}}_{\delta} H^{\mathrm{C}}_{\gamma\, b\, a}$ as well. As will become clear in the explicit discussion, $H^{\mathrm{C}}_{\gamma\, b\, a}|$ will be related to the helicity 1/2 gravigini components of the multiplet. Therefore, the soldering operation implies that the graviphoton field strength appears in a well defined way in the supersymmetry transformation of both the gravitini fields and the helicity 1/2 components.

On the other hand, the relation between $H_{\mathbf{u}\, b\, a}$ and $T_{cb}{}^{\mathbf{u}}$ implies the appearance of a graviphoton Chern–Simons form in $H_{c\, b\, a}|$, as we will show now. Note that this is a property which appears at the component field level. Using the double bar projection $E^{\mathcal{A}}|| = e^{\mathcal{A}}$ equivalently in the coordinate and the covariant frame basis, we obtain

$$
H|| \;=\; \frac{1}{3!} dx^m dx^n dx^k \partial_k b_{nm} \tag{3.7}
$$

$$
\begin{aligned}
H|| \;=\;& \frac{1}{3!} e^a e^b e^c H_{c\, b\, a}| + \frac{1}{2} e^a e^b e^{\mathbf{u}} H_{\mathbf{u}\, b\, a}| \\
&+ \frac{1}{2} e^a e^b e^{\gamma}_{\mathrm{C}} H^{\mathrm{C}}_{\gamma\, b\, a}| + \frac{1}{2} e^a e^b e^{\mathrm{C}}_{\dot\gamma} H^{\dot\gamma}_{\mathrm{C}\, b\, a}| \\
&+ \frac{1}{2} e^{\mathbf{u}} e^{\beta}_{\mathrm{B}} e^{\gamma}_{\mathrm{C}} H^{\mathrm{C\,B}}_{\gamma\,\beta\,\mathbf{u}}| + e^a e^{\beta}_{\mathrm{B}} e^{\mathrm{C}}_{\dot\gamma} H^{\dot\gamma\,\mathrm{B}}_{\mathrm{C}\,\beta\, a}| + \frac{1}{2} e^{\mathbf{u}} e^{\mathrm{B}}_{\dot\beta} e^{\mathrm{C}}_{\dot\gamma} H^{\dot\gamma\,\dot\beta}_{\mathrm{C\,B}\,\mathbf{u}}| \\
&+ e^a e^{\beta}_{\mathrm{B}} e^{\mathbf{u}} H_{\mathbf{u}\,\beta\, a}{}^{\mathrm{B}}| + e^a e^{\mathrm{B}}_{\dot\beta} e^{\mathbf{u}} H_{\mathbf{u}\,\mathrm{B}\, a}{}^{\dot\beta}|\ .
\end{aligned} \tag{3.8}
$$

The term giving rise to the Chern–Simons coupling of the graviphotons is

$$
e^a e^b e^{\mathbf{u}} H_{\mathbf{u}\, b\, a}| \;=\; dx^m dx^n dx^l e_l{}^a e_n{}^b v_m{}^{\mathbf{u}}\, H_{\mathbf{u}ba}|\ , \tag{3.9}
$$

due to the fact that $e^{\mathbf{u}} = dx^m v_m{}^{\mathbf{u}}$ and that $H_{\mathbf{u}ba}$ is related to the graviphoton field strength according to the previous discussion.

The qualitative discussion in this section should give a flavor of the conceptual basis of the central charge superspace and its impact on the structure of the $N = 4$ supergravity multiplet. In the sequel we provide some of the technical details in a compact way.

4 Superspace and the N–T multiplet

As we have seen qualitatively, the soldering mechanism intertwining the 2–form with the supergravity geometry is essential for the superspace description of $N = 4$ supergravity with an antisymmetric tensor. Let us now have a somewhat closer look at the superspace geometry pertaining to the N–T multiplet.

As has become common usage we shall exploit the consequences of torsion and 3–form curl constraints in analysing the respective Bianchi identities in the order of increasing engineering dimension (in units of +1 for space–time and +1/2 for spinor derivatives) normalized such that the torsion and 3–form curl components displayed in (3.2) have dimension 0.

We shall however refrain here from a distinction between *constraints* and *consequences of constraints* and simply display the properties of torsion and 3–form curl suitable for the description of the N–T multiplet. We stress first of all that the structure group is taken to be Lorentz$\times SU(4)$ in the conventional superspace sector, and trivial in the central charge sector.

Vierbein and gravitini, as well as the spin connection being identified in the usual way, we shall illustrate in the following the salient superspace structures relevant for the description of the remaining components of the N–T multiplet.

- *Graviscalar and gravigini*

As to the systematic discussion of $H_{\mathcal{CBA}}$, we note that the coefficients at dimension -1/2 (spinor indices only) all vanish. At dimension 0, see eq. (3.2), we parametrize

$$L \ = \ e^{2\phi} \ . \tag{4.1}$$

The lowest component of the real superfield ϕ will be identified as the scalar component field of the N–T multiplet.

This real superfield will appear in the converters $T^{[\mathrm{CB}]\mathbf{u}}$, $T_{[\mathrm{CB}]}{}^{\mathbf{u}}$ as well,

$$T^{[\mathrm{CB}]\mathbf{u}} \ = \ e^{\phi}\, a^{[\mathrm{CB}]\mathbf{u}} \ , \qquad T_{[\mathrm{CB}]}{}^{\mathbf{u}} \ = \ e^{\phi}\, a_{[\mathrm{CB}]}{}^{\mathbf{u}} \ . \tag{4.2}$$

Likewise, in the H–sector, we identify

$$H_{\mathbf{u}}{}^{[\mathrm{CB}]} \ = \ e^{\phi}\, m_{\mathbf{u}}{}^{[\mathrm{CB}]} \ , \qquad H_{\mathbf{u}[\mathrm{CB}]} \ = \ e^{\phi}\, m_{\mathbf{u}[\mathrm{CB}]} \ . \tag{4.3}$$

The constant matrices $a^{[\mathrm{CB}]\mathbf{u}}$, $a_{[\mathrm{CB}]}{}^{\mathbf{u}}$ and $m_{\mathbf{u}}{}^{[\mathrm{CB}]}$, $m_{\mathbf{u}[\mathrm{CB}]}$ are supposed to satisfy the relations[2]

$$a^{[\mathrm{DC}]\mathbf{u}}\, m_{\mathbf{u}}{}^{[\mathrm{BA}]} \;=\; 8\,\epsilon^{\mathrm{DCBA}} \;, \qquad a_{[\mathrm{DC}]}{}^{\mathbf{u}}\, m_{\mathbf{u}[\mathrm{BA}]} \;=\; 8\,\epsilon_{\mathrm{DCBA}} \;, \tag{4.4}$$

$$a^{[\mathrm{DC}]\mathbf{u}}\, m_{\mathbf{u}[\mathrm{BA}]} + a_{[\mathrm{BA}]}{}^{\mathbf{u}}\, m_{\mathbf{u}}{}^{[\mathrm{DC}]} \;=\; 16\,\delta^{\mathrm{DC}}_{\mathrm{BA}} \;. \tag{4.5}$$

Furthermore, we impose the self–duality relations[3]

$$a^{[\mathrm{DC}]\mathbf{u}} \;=\; \frac{1}{2}\,\epsilon^{\mathrm{DCBA}} a_{[\mathrm{BA}]}{}^{\mathbf{u}} \;, \qquad m_{\mathbf{u}}{}^{[\mathrm{BA}]} \;=\; \frac{1}{2}\, m_{\mathbf{u}[\mathrm{DC}]}\epsilon^{\mathrm{DCBA}} \;. \tag{4.6}$$

As a consequence, the second relation above becomes

$$a^{[\mathrm{DC}]\mathbf{u}}\, m_{\mathbf{u}[\mathrm{BA}]} \;=\; a_{[\mathrm{BA}]}{}^{\mathbf{u}}\, m_{\mathbf{u}}{}^{[\mathrm{DC}]} \;=\; 8\,\delta^{\mathrm{DC}}_{\mathrm{BA}} \;. \tag{4.7}$$

For consistency, we have as well

$$m_{\mathbf{u}[\mathrm{FE}]}\, a^{[\mathrm{FE}]\mathbf{v}} \;=\; 16\,\delta_{\mathbf{u}}^{\mathbf{v}} \;. \tag{4.8}$$

As a consequence, any object $X_{\mathbf{u}}$ or $Y^{\mathbf{u}}$, once converted to the $SU(4)$ basis,

$$X^{[\mathrm{DC}]} \;=\; a^{[\mathrm{DC}]\mathbf{u}}\, X_{\mathbf{u}} \;, \qquad X_{[\mathrm{BA}]} \;=\; a_{[\mathrm{BA}]}{}^{\mathbf{u}} X_{\mathbf{u}} \;, \tag{4.9}$$

$$Y_{[\mathrm{DC}]} \;=\; Y^{\mathbf{u}}\, m_{\mathbf{u}[\mathrm{DC}]} \;, \qquad Y^{[\mathrm{BA}]} \;=\; Y^{\mathbf{u}}\, m_{\mathbf{u}}{}^{[\mathrm{BA}]} \;, \tag{4.10}$$

satisfies the self–duality relations

$$X^{[\mathrm{DC}]} \;=\; \frac{1}{2}\,\epsilon^{\mathrm{DCBA}} X_{[\mathrm{BA}]} \;, \qquad Y_{[\mathrm{BA}]} \;=\; \frac{1}{2}\, Y^{[\mathrm{DC}]}\epsilon_{\mathrm{DCBA}} \;, \tag{4.11}$$

similar to the reality conditions employed in the description of the $N = 4$ Yang–Mills theory [16].

Whereas the real graviscalar is identified as the lowest component of the *graviscalar superfield* ϕ, its covariant fermionic partner, the gravigino (in reference to the gaugino in supersymmetric Yang–Mills theory), appears as lowest component of the *gravigino superfield* $T^{[\mathrm{CBA}]}{}_{\dot\alpha}$, $T_{[\mathrm{CBA}]}{}^{\alpha}$. Here, it is costumary to parametrize

$$T^{[\mathrm{CBA}]}{}_{\dot\alpha} \;=\; \bar\lambda_{\dot\alpha \mathrm{D}}\, \epsilon^{\mathrm{DCBA}} \;, \qquad T_{[\mathrm{CBA}]}{}^{\alpha} \;=\; \lambda^{\alpha \mathrm{D}}\, \epsilon_{\mathrm{DCBA}} \;. \tag{4.12}$$

Recall that, at dimension 1/2, there is quite a large number of torsion and 3–form curl coefficients. Injecting the structure of torsion and 3–form at dimensions -1/2 and 0 into the dimension 1/2 Bianchi identities, together with certain conventional constraints, implies that all the non–vanishing coefficients are expressed in terms of the gravigino superfields. The coefficients given in terms of $\lambda^{\alpha \mathrm{A}}$ are

$$T_{\gamma\,\mathrm{B}\,\dot\alpha}^{\mathrm{C}\,\dot\beta\,\mathrm{A}} \;=\; -\frac{1}{4}\,\delta^{\dot\beta}_{\dot\alpha}\delta^{\mathrm{A}}_{\mathrm{B}}\,\lambda^{\mathrm{C}}_{\gamma} \;, \qquad T_{\gamma\,\beta\,\alpha}^{\mathrm{C\,B\,A}} \;=\; \frac{1}{4}(\delta^{\alpha}_{\beta}\delta^{\mathrm{B}}_{\mathrm{A}}\,\lambda^{\mathrm{C}}_{\gamma} + \delta^{\alpha}_{\gamma}\delta^{\mathrm{C}}_{\mathrm{A}}\,\lambda^{\mathrm{B}}_{\beta}) \;,$$

$$\text{and} \qquad H^{\mathrm{C}}_{\gamma\,b\,a} \;=\; -2(\sigma_{ba})_{\gamma}{}^{\varphi}\,\lambda^{\mathrm{C}}_{\varphi}\, e^{2\phi} \;, \tag{4.13}$$

[2]We have fixed certain *a priori* free parameters such as to fit with the N–T multiplet, other possibilities are studied in [15].

[3]In fact, these relations may be obtained as a consequence of the particular structure of the superspace geometry [15].

in the conventional superspace sector, whereas in the central charge sector we find

$$T_{c\,\mathrm{B}}{}^{\dot\beta\,\mathbf{u}} \;=\; \frac{i}{4} e^{\phi}\, \bar\sigma_c^{\dot\beta\varphi}\, \lambda_\varphi^{\mathrm{F}}\, a_{[\mathrm{FB}]}{}^{\mathbf{u}} \,, \qquad H_{\mathbf{u}\,b\,\mathrm{A}}^{\;\;\;\;\dot\alpha} \;=\; \frac{i}{4} e^{\phi}\, \bar\sigma_b^{\dot\alpha\varphi}\, \lambda_\varphi^{\mathrm{F}}\, m_{\mathbf{u}[\mathrm{FA}]} \,. \tag{4.14}$$

It is interesting to note that these two expressions are related through

$$H_{\mathbf{u}\,b\,\mathrm{A}}^{\;\;\;\;\dot\alpha} \;=\; T_{b\,\mathrm{A}}{}^{\dot\alpha\,\mathbf{v}}\, g_{\mathbf{vu}} \qquad \text{with} \quad g_{\mathbf{vu}} \;=\; \frac{1}{32}\, m_{\mathbf{v}}{}^{[\mathrm{DC}]}\, m_{\mathbf{u}}{}^{[\mathrm{BA}]}\, \epsilon_{\mathrm{DCBA}} \,, \tag{4.15}$$

or, conversely

$$T_{b\,\mathrm{A}}{}^{\dot\alpha\,\mathbf{u}} \;=\; g^{\mathbf{uv}}\, H_{\mathbf{v}\,b\,\mathrm{A}}^{\;\;\;\;\dot\alpha} \qquad \text{with} \quad g^{\mathbf{uv}} \;=\; \frac{1}{32}\, a_{[\mathrm{DC}]}{}^{\mathbf{u}} a_{[\mathrm{BA}]}{}^{\mathbf{v}} \epsilon^{\mathrm{DCBA}} \,. \tag{4.16}$$

Of course, the metric $g_{\mathbf{uv}}$ in the central charge basis satisfies

$$g_{\mathbf{uw}} g^{\mathbf{wv}} \;=\; \delta_{\mathbf{u}}^{\mathbf{v}} \,. \tag{4.17}$$

It should be clear that in deriving these results the superspace soldering mechanism has already been at work. Likewise, for the $\bar\lambda_{\dot\alpha\mathrm{A}}$ dependent coefficients, we find

$$T_{\mathrm{C}\,\beta\,\mathrm{A}}^{\dot\gamma\,\mathrm{B}\,\alpha} \;=\; -\frac{1}{4}\, \delta_\beta^\alpha \delta_{\mathrm{A}}^{\mathrm{B}}\, \bar\lambda_{\mathrm{C}}^{\dot\gamma} \,, \qquad T_{\mathrm{C}\,\mathrm{B}\,\dot\alpha}^{\dot\gamma\,\dot\beta\,\mathrm{A}} \;=\; \frac{1}{4} (\delta_{\dot\alpha}^{\dot\beta} \delta_{\mathrm{B}}^{\mathrm{A}}\, \bar\lambda_{\mathrm{C}}^{\dot\gamma} + \delta_{\dot\alpha}^{\dot\gamma} \delta_{\mathrm{C}}^{\mathrm{A}}\, \bar\lambda_{\mathrm{B}}^{\dot\beta}) \,,$$
$$\text{and} \qquad H_{\mathrm{C}\,b\,a}^{\dot\gamma} \;=\; -2 (\bar\sigma_{ba})^{\dot\gamma}{}_{\dot\varphi}\, \bar\lambda_{\mathrm{C}}^{\dot\varphi}\, e^{2\phi} \,, \tag{4.18}$$

in the conventional sector, and

$$T_{c\,\beta}{}^{\mathrm{B}\,\mathbf{u}} \;=\; \frac{i}{4} e^{\phi}\, (\bar\sigma_c)_{\beta\dot\varphi}\, \bar\lambda_{\mathrm{F}}^{\dot\varphi}\, a^{\mathbf{u}[\mathrm{FB}]} \,, \qquad H_{\mathbf{u}\,b\,\alpha}^{\;\;\;\;\mathrm{A}} \;=\; \frac{i}{4} e^{\phi}\, (\sigma_b)_{\alpha\dot\varphi}\, \bar\lambda_{\mathrm{F}}^{\dot\varphi}\, m_{\mathbf{u}}{}^{[\mathrm{FA}]} \,, \tag{4.19}$$

in the central charge sector. As before, $H_{\mathbf{u}\,b\,\alpha}^{\;\;\;\;\mathrm{A}}$ and $T_{b\,\alpha}{}^{\mathrm{A}\,\mathbf{u}}$ are related through the metric in the central charge basis.

Finally, the dimension 1/2 Bianchi identities imply

$$\lambda_\alpha^{\mathrm{A}} \;=\; -2\, \mathcal{D}_\alpha^{\mathrm{A}} \phi \,, \qquad \bar\lambda_{\mathrm{A}}^{\dot\alpha} \;=\; -2\, \bar{\mathcal{D}}_{\mathrm{A}}^{\dot\alpha} \phi \,. \tag{4.20}$$

We have thus completely exhausted the information contained in the dimension 1/2 Bianchi identities.

- *Graviphotons*

Central charge superspace has been conceived as a concise framework to describe graviphotons as messengers of local central charge transformations based on a sound geometric basis. The covariant fieldstrength of the graviphotons is identified in the superspace torsion coefficient $T_{cb}{}^{\mathbf{u}}$.

As anticipated in the introduction, the superspace soldering procedure implies that certain torsion and 3–form curl coefficients will be expressed in terms of the graviphoton torsion, intertwining supergravity and the 2–form geometries in an intricate way. As to the torsion coefficients, at dimension 1 we find

$$T_{\gamma\, b\, \dot\alpha}^{\mathrm{C}\ \ \mathrm{A}} \;=\; \frac{i}{8}\, e^{-\phi}\, (\sigma_b \bar\sigma^{dc})_{\gamma\, \dot\alpha}\, F_{dc}{}^{[\mathrm{CA}]} \ , \tag{4.21}$$

$$T^{\dot\gamma\ \ \alpha}_{\mathrm{C}\, b\, \mathrm{A}} \;=\; \frac{i}{8}\, e^{-\phi}\, (\bar\sigma_b \sigma^{dc})^{\dot\gamma\, \alpha}\, F_{dc\, [\mathrm{CA}]} \ , \tag{4.22}$$

where we have introduced the notation

$$F_{dc}{}^{[\mathrm{BA}]} \;=\; T_{dc}{}^{\mathbf{u}} m_{\mathbf{u}}{}^{[\mathrm{BA}]} \ , \qquad F_{dc\, [\mathrm{BA}]} \;=\; T_{dc}{}^{\mathbf{u}} m_{\mathbf{u}\, [\mathrm{BA}]} \ . \tag{4.23}$$

These relations determine the appearance of the graviphotons in the supersymmetry transformations laws of the gravitini.

As a consequence of the superspace soldering, the graviphotons appear in the 2–form geometry in the coefficients

$$H_{\mathbf{u}\, b\, a} \;=\; T_{b\, a}{}^{\mathbf{v}}\, g_{\mathbf{vu}} \ . \tag{4.24}$$

As we have already stressed, this is responsible for the appearance of graviphoton Chern–Simons forms in the supercovariant curl of the antisymmetric tensor. Finally, the graviphoton fieldstrength appears also in the second spinor derivative of the graviscalar superfield such that

$$\mathcal{D}^{\mathrm{B}}_{\beta} \mathcal{D}^{\mathrm{A}}_{\alpha} \phi \;=\; -\frac{1}{4}\, e^{-\phi} (\sigma^{ba} \epsilon)_{\beta\alpha}\, F_{ba}{}^{[\mathrm{BA}]} - \frac{3}{8}\, \lambda^{\mathrm{B}}_{\beta} \lambda^{\mathrm{A}}_{\alpha} \ , \tag{4.25}$$

$$\mathcal{D}^{\dot\beta}_{\mathrm{B}} \mathcal{D}^{\dot\alpha}_{\mathrm{A}} \phi \;=\; -\frac{1}{4}\, e^{-\phi} (\bar\sigma^{ba} \epsilon)^{\dot\beta\dot\alpha}\, F_{ba\, [\mathrm{BA}]} - \frac{3}{8}\, \bar\lambda^{\dot\beta}_{\mathrm{B}} \bar\lambda^{\dot\alpha}_{\mathrm{A}} \ . \tag{4.26}$$

This last relation illustrates how $F_{ba}{}^{[\mathrm{BA}]}$ appears in the supersymmetry transformations of the gravigini.

- *Antisymmetric tensor*

As to the antisymmetric tensor b_{mn}, the superspace soldering mechanism allows to identify its covariant 3–form fieldstrength H_{cba} in certain torsion coefficients. More explicitly, as a consequence of our constraints, the Bianchi identities give rise to

$$T_{\gamma\, b\, \mathrm{A}}^{\mathrm{C}\ \ \alpha} \;=\; -\frac{1}{4}\, \delta^{\mathrm{C}}_{\mathrm{A}}\, \delta^{\alpha}_{\gamma}\, H^{*}_{b} e^{-2\phi} - \frac{i}{8} \left(\delta^{\mathrm{C}}_{\mathrm{A}}\, \lambda^{\mathrm{F}}_{\gamma} \bar\lambda_{\dot\alpha \mathrm{F}}\, \bar\sigma_b^{\dot\alpha\alpha} + 2\, \lambda^{\alpha\mathrm{C}} \sigma_{b\, \gamma\dot\gamma} \bar\lambda^{\dot\gamma}_{\mathrm{A}} \right) \ , \tag{4.27}$$

$$T^{\dot\gamma\ \ \mathrm{A}}_{\mathrm{C}\, b\, \dot\alpha} \;=\; -\frac{1}{4}\, \delta^{\mathrm{A}}_{\mathrm{C}}\, \delta^{\dot\gamma}_{\dot\alpha}\, H^{*}_{b} e^{-2\phi} - \frac{i}{8} \left(\delta^{\mathrm{A}}_{\mathrm{C}}\, \bar\lambda^{\dot\gamma}_{\mathrm{F}} \lambda^{\alpha\mathrm{F}} \sigma_{b\, \alpha\dot\alpha} + 2\, \bar\lambda_{\dot\alpha \mathrm{C}} \bar\sigma_b^{\dot\gamma\gamma} \lambda^{\mathrm{A}}_{\gamma} \right) \ , \tag{4.28}$$

with the dual tensor defined as

$$H^{*}_{d} = \frac{1}{3!}\, \epsilon_{dcba}\, H^{cba} \ . \tag{4.29}$$

This indicates how the antisymmetric tensor will appear in the supersymmetry transformation of the gravitino, after suitable identification and projection to lowest components in (2.7).

In turn, the same Bianchi identity leads to the relation

$$\mathcal{D}^{\dot\beta}_{\mathrm{B}} \mathcal{D}^{\mathrm{A}}_{\alpha} \phi \;=\; i \delta^{\mathrm{A}}_{\mathrm{B}} (\sigma^a \epsilon)_{\alpha}{}^{\dot\beta} \mathcal{D}_a \phi + \frac{1}{2} e^{-2\phi} \delta^{\mathrm{A}}_{\mathrm{B}} (\sigma^a \epsilon)_{\alpha}{}^{\dot\beta} H^{*}_{a} - \frac{3}{8} \left(\lambda^{\mathrm{A}}_{\alpha} \bar\lambda^{\dot\beta}_{\mathrm{B}} - 2 \delta^{\mathrm{A}}_{\mathrm{B}} \lambda^{\mathrm{F}}_{\alpha} \bar\lambda^{\dot\beta}_{\mathrm{F}} \right) \ , \tag{4.30}$$

illustrating how the antisymmetric tensor appears in the supersymmetry transformation of the gravigini.

5 Component field transformations

As to the component fields of the N–T multiplet, the graviton, gravitini and graviphotons are identified in the superspace frame $E^{\mathcal{A}}$ (1.1), the antisymmetric tensor in the superspace 2–form B (1.2), whereas the real scalar and the helicity 1/2 gravigini fields are found as the lowest components of the superfields ϕ, $\lambda_\alpha^{\mathrm{A}}$, $\bar{\lambda}^{\dot{\alpha}}_{\mathrm{A}}$. We shall use the same symbols to denote the component fields, *i.e.*

$$\phi| \;=\; \phi \,, \qquad \lambda_\alpha^{\mathrm{A}}| \;=\; \lambda_\alpha^{\mathrm{A}} \,, \qquad \bar{\lambda}^{\dot{\alpha}}_{\mathrm{A}}| \;=\; \bar{\lambda}^{\dot{\alpha}}_{\mathrm{A}} \,. \tag{5.1}$$

The Wess-Zumino transformations of graviton, gravitini and graviphotons can then be read off from suitable projections of the superfield equation (2.7). For the frame in space-time one obtains the usual result

$$\delta_{\mathrm{WZ}}\, e_m{}^a \;=\; -i\, \psi_m{}^\beta_{\mathrm{B}} (\sigma^a \epsilon)_\beta{}^{\dot{\beta}}\, \bar{\zeta}^{\mathrm{B}}_{\dot{\beta}} - i\, \bar{\psi}_m{}^{\mathrm{B}}_{\dot{\beta}} (\bar{\sigma}^a \epsilon)^{\dot{\beta}}{}_\beta\, \zeta^\beta_{\mathrm{B}} \,, \tag{5.2}$$

whereas the transformations for the Rarita-Schwinger fields are

$$\begin{aligned}
\delta_{\mathrm{WZ}}\, \psi_m{}^\alpha_{\mathrm{A}} \;=\;& 2\,\mathcal{D}_m\, \zeta^\alpha_{\mathrm{A}} - \frac{i}{4} e^{-\phi} e_m{}^b (\bar{\zeta}^{\mathrm{B}} \bar{\sigma}^a)^\alpha F^+_{ba\,[\mathrm{BA}]}| - \frac{1}{2} e^{-2\phi} e_m{}^b \zeta^\alpha_{\mathrm{A}} H^*_b| \\
&+ \frac{1}{4}\, \psi_m{}^\alpha_{\mathrm{A}} (\zeta \cdot \lambda - \bar{\zeta} \cdot \bar{\lambda}) + \frac{1}{4} (\psi_m \cdot \lambda - \bar{\psi}_m \cdot \bar{\lambda})\, \zeta^\alpha_{\mathrm{A}} \\
&+ \frac{i}{2} \lambda^{\alpha\,\mathrm{B}} (\zeta_{\mathrm{B}} \sigma_m \bar{\lambda}_{\mathrm{A}} - 2i \bar{\psi}_m{}^{\mathrm{D}} \bar{\zeta}^{\mathrm{C}} \varepsilon_{\mathrm{DCBA}}) - \frac{i}{4} (\zeta_{\mathrm{A}} \lambda^{\mathrm{B}}) (\bar{\lambda}_{\mathrm{B}} \bar{\sigma}_m)^\alpha \,,
\end{aligned} \tag{5.3}$$

$$\begin{aligned}
\delta_{\mathrm{WZ}}\, \bar{\psi}_m{}^{\mathrm{A}}_{\dot{\alpha}} \;=\;& 2\,\mathcal{D}_m\, \bar{\zeta}^{\mathrm{A}}_{\dot{\alpha}} - \frac{i}{4} e^{-\phi} e_m{}^b (\zeta_{\mathrm{B}} \sigma^a)_{\dot{\alpha}} F^{-[\mathrm{BA}]}_{ba}| + \frac{1}{2} e^{-2\phi} e_m{}^b \bar{\zeta}^{\mathrm{A}}_{\dot{\alpha}} H^*_b| \\
&- \frac{1}{4}\, \bar{\psi}_m{}^{\mathrm{A}}_{\dot{\alpha}} (\zeta \cdot \lambda - \bar{\zeta} \cdot \bar{\lambda}) - \frac{1}{4} (\psi_m \cdot \lambda - \bar{\psi}_m \cdot \bar{\lambda})\, \bar{\zeta}^{\mathrm{A}}_{\dot{\alpha}} \\
&+ \frac{i}{2} \bar{\lambda}_{\dot{\alpha}\mathrm{B}} (\bar{\zeta}^{\mathrm{B}} \bar{\sigma}_m \lambda^{\mathrm{A}} - 2i \psi_{m\mathrm{D}} \zeta_{\mathrm{C}} \varepsilon^{\mathrm{DCBA}}) + \frac{i}{4} (\bar{\zeta}^{\mathrm{A}} \bar{\lambda}_{\mathrm{B}}) (\lambda^{\mathrm{B}} \sigma_m)_{\dot{\alpha}} \,,
\end{aligned} \tag{5.4}$$

with definitions $F^\pm_{ba} = F_{ba} \pm \frac{i}{2} F^{dc}\, \varepsilon_{dcba}$ for the selfdual and antiselfdual components of the central charge fieldstrength and $\zeta \cdot \lambda = \zeta^\alpha_{\mathrm{A}} \lambda^{\mathrm{A}}_\alpha$, $\bar{\zeta} \cdot \bar{\lambda} = \bar{\zeta}^{\mathrm{A}}_{\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}}_{\mathrm{A}}$ for summation conventions. Observe also the appearance of the field strength of the antisymmetric tensor, which is due to the soldering mechanism. As the supervielbein contains the graviphotons as well, relation (2.7) implies

$$\delta_{\mathrm{WZ}} V_m{}^{[\mathrm{BA}]} \;=\; -2 e^\phi (i \zeta_{\mathrm{D}} \sigma_m \bar{\lambda}_{\mathrm{C}} - 2 \zeta_{\mathrm{D}} \psi_{m\,\mathrm{C}})\, \epsilon^{\mathrm{DCBA}} - 2 e^\phi (i \bar{\zeta}^{\mathrm{D}} \bar{\sigma}_m \lambda^{\mathrm{C}} - 2 \bar{\zeta}^{\mathrm{D}} \bar{\psi}_m{}^{\mathrm{C}})\, \delta^{\mathrm{BA}}_{\mathrm{DC}} \,. \tag{5.5}$$

As to the antisymmetric tensor one applies (2.8), with the result

$$\begin{aligned}
\delta_{\mathrm{WZ}}\, b_{mn} \;=\;& -2\, e^{2\phi} (\zeta_{\mathrm{A}} \sigma_{mn} \lambda^{\mathrm{A}}) - 2\, e^{2\phi} (\bar{\zeta}^{\mathrm{A}} \bar{\sigma}_{mn} \bar{\lambda}_{\mathrm{A}}) - i (\bar{\psi}_{[m}{}^{\mathrm{A}} \bar{\sigma}_{n]} \zeta_{\mathrm{A}}) - i (\psi_{[m\mathrm{A}} \sigma_{n]} \bar{\zeta}^{\mathrm{A}}) \\
&+ \frac{1}{2} V_{[m}{}^{[\mathrm{BA}]} (\psi_{n]\mathrm{A}} \zeta_{\mathrm{B}}) - \frac{i}{4} e^\phi\, V_{[m}{}^{[\mathrm{BA}]} (\zeta_{\mathrm{A}} \sigma_{n]} \bar{\lambda}_{\mathrm{B}}) \\
&+ \frac{1}{2} V_{[m\,[\mathrm{BA}]} (\bar{\psi}_{n]}{}^{\mathrm{A}} \bar{\zeta}^{\mathrm{B}}) - \frac{i}{4} e^\phi\, V_{[m\,[\mathrm{BA}]} (\bar{\zeta}^{\mathrm{A}} \bar{\sigma}_{n]} \lambda^{\mathrm{B}}) \,,
\end{aligned} \tag{5.6}$$

where the antisymmetrization convention $X_{[m} Y_{n]} = X_m Y_n - X_n Y_m$ for any vectors X_m and Y_n is used. Finally, the transformations of graviscalar and gravigini are derived in the usual way as well, one obtains

$$\delta_{\mathrm{WZ}}\, \phi \;=\; \zeta^\alpha_{\mathrm{A}} \lambda^{\mathrm{A}}_\alpha + \bar{\zeta}^{\mathrm{A}}_{\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}}_{\mathrm{A}} \,, \tag{5.7}$$

$$
\begin{aligned}
\delta_{\mathrm{WZ}}\,\lambda_\alpha^{\mathrm{A}} &= -2i(\bar\zeta^{\mathrm{A}}\bar\sigma^a\epsilon)_\alpha\,\mathcal{D}_a\phi| + \frac{1}{2}e^{-\phi}(\zeta_{\mathrm{B}}\sigma^{ba}\epsilon)_\alpha\,F_{ba}{}^{[\mathrm{BA}]}| - e^{-2\phi}(\bar\zeta^{\mathrm{A}}\bar\sigma^a\epsilon)_\alpha H^*_a| \\
&\quad +3(\zeta\cdot\lambda - \bar\zeta\cdot\bar\lambda)\lambda_\alpha^{\mathrm{A}} + 6(\bar\zeta^{\mathrm{A}}\bar\lambda_{\mathrm{B}})\lambda_\alpha^{\mathrm{B}}\ , \qquad (5.8)
\end{aligned}
$$

$$
\begin{aligned}
\delta_{\mathrm{WZ}}\,\bar\lambda^{\dot\alpha}_{\mathrm{A}} &= -2i(\bar\zeta^{\mathrm{A}}\bar\sigma^a\epsilon)_\alpha\,\mathcal{D}_a\phi| + \frac{1}{2}e^{-\phi}(\bar\zeta^{\mathrm{B}}\bar\sigma^{ba}\epsilon)^{\dot\alpha}F_{ba\,[\mathrm{BA}]}| + e^{-2\phi}(\zeta_{\mathrm{A}}\sigma^a\epsilon)^{\dot\alpha}H^*_a| \\
&\quad -3(\zeta\cdot\lambda - \bar\zeta\cdot\bar\lambda)\bar\lambda^{\dot\alpha}_{\mathrm{A}} + 6(\zeta_{\mathrm{A}}\lambda^{\mathrm{B}})\bar\lambda^{\dot\alpha}_{\mathrm{B}}\ . \qquad (5.9)
\end{aligned}
$$

In these expressions the component field supercovariant field strength and derivatives are defined as

$$
\begin{aligned}
F_{ba}{}^{[\mathrm{BA}]}| &= e_b{}^n e_a{}^m \partial_{[n}V_{m]}{}^{[\mathrm{BA}]} - 2e_b{}^n e_a{}^m\, e^\phi\left(\psi_{m\,\mathrm{D}}\psi_{n\,\mathrm{C}}\epsilon^{\mathrm{DCBA}} + \bar\psi_m{}^{\mathrm{D}}\bar\psi_n{}^{\mathrm{C}}\delta^{\mathrm{BA}}_{\mathrm{DC}}\right) \\
&\quad +i\, e_b{}^n e_a{}^m\, e^\phi\left(\psi_{[m\,\mathrm{D}}\sigma_{n]}\bar\lambda_{\mathrm{C}}\epsilon^{\mathrm{DCBA}} + \bar\psi_{[m}{}^{\mathrm{D}}\bar\sigma_{n]}\lambda^{\mathrm{C}}\delta^{\mathrm{BA}}_{\mathrm{DC}}\right)\ , \qquad (5.10)
\end{aligned}
$$

for the graviphotons,

$$
\begin{aligned}
H^{*b}| &= \frac{1}{2}\left(\partial_m b_{lk} - \frac{1}{8}V_{m[\mathrm{BA}]}\partial_l V_k{}^{[\mathrm{BA}]}\right)\epsilon^{klmn}e_n{}^b \\
&\quad \frac{1}{2}e^{2\phi}\left(i\psi_{m\,\mathrm{B}}\sigma_l\bar\psi_k{}^{\mathrm{B}} + \psi_{m\,\mathrm{C}}\sigma_{lk}\lambda^{\mathrm{C}} + \bar\psi_m{}^{\mathrm{C}}\bar\sigma_{lk}\bar\lambda_{\mathrm{C}}\right)\epsilon^{klmn}e_n{}^b\ , \qquad (5.11)
\end{aligned}
$$

for the antisymmetric tensor and

$$
\mathcal{D}_a\phi| = e_a{}^m\partial_m\phi + \frac{1}{4}e_a{}^m(\psi_m\cdot\lambda - \bar\psi_m\cdot\bar\lambda)\ , \qquad (5.12)
$$

for the graviscalar.

As already stressed, central charge transformations are special cases of supergravity transformations as well. The central charge transformations of component fields are defined in terms of local parameters $\xi^{\mathcal{A}}| = (0,0,0,\zeta^{\mathbf{u}})$. Assuming that the dependence of the component fields on the central charge directions is trivial, it is immediate to see that only the graviphotons and the antisymmetric tensor have non–trivial central charge transformations. Using again (2.7) and (2.8) with suitable component field projections, one derives the transformation laws

$$
\delta_{\mathbf{cc}}\,V_m{}^{[\mathrm{BA}]} = \partial_m\zeta^{[\mathrm{BA}]}\ , \qquad (5.13)
$$

for the graviphotons and

$$
\delta_{\mathbf{cc}}\,b_{mn} = \frac{1}{16}\left(\partial_m V_n{}^{[\mathrm{BA}]} - \partial_n V_m{}^{[\mathrm{BA}]}\right)\zeta_{[\mathrm{BA}]}\ , \qquad (5.14)
$$

for the antisymmetric tensor with the obvious notation $\zeta^{[\mathrm{BA}]} = \zeta^{\mathbf{u}}m_{\mathbf{u}}{}^{[\mathrm{BA}]}$.

6 Conclusion

We have shown that the identification of the graviphotons in $E_{\mathcal{M}}{}^{\mathcal{A}}$, the frame of superspace [10], together with the soldering mechanism, which provides an intimate relation between gravity and 2–form geometries, clarifies a number of points in the description of the N–T supergravity, *i.e.* $N = 4$ supergravity with an antisymmetric tensor gauge field.

One of the important points is that the graviscalar appears both in the gravity and in the 2–form sectors, as given in (4.2) and (4.3). The symbols $a^{\mathbf{u}[\mathrm{DC}]}$ and $m_{\mathbf{u}}{}^{[\mathrm{DC}]}$ which appear there serve to convert between the central charge and the $SU(4)$ bases. Moreover, the particular constraint structure in superspace suggests selfduality relations

$$a^{\mathbf{u}[\mathrm{DC}]} = \frac{1}{2}\epsilon^{\mathrm{DCBA}} a^{\mathbf{u}}{}_{[\mathrm{BA}]} , \qquad m_{\mathbf{u}}{}^{[\mathrm{DC}]} = \frac{1}{2}\epsilon^{\mathrm{DCBA}} m_{\mathbf{u}[\mathrm{BA}]} . \tag{6.1}$$

As a consequence, quantities as the graviphoton field strength

$$F_{\mathcal{DC}}{}^{[\mathrm{BA}]} = T_{\mathcal{DC}}{}^{\mathbf{u}} m_{\mathbf{u}}{}^{[\mathrm{BA}]} = a^{[\mathrm{BA}]\mathbf{u}} H_{\mathbf{u}\mathcal{DC}} , \tag{6.2}$$

are selfdual in the same sense. The appearance of the graviphoton field strength in the 2–form sector explains in a neat way the presence of the graviphoton Chern–Simons form in the supercovariant curl $H_{cba}|$ of the antisymmetric tensor. It should be interesting to reinvestigate the coupling of $N = 4$ matter to $N = 4$ supergravity [17], [18] in the geometric framework set up here as well.

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